Signs of complete square

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We all know:

1. $(a+b)^2 = a^2 + 2ab + b^2$

2. $(a-b)^2 = a^2 - 2ab + b^2$

But we can observe that :

3. $(-a-b)^2 = [-(a+b)]^2 = (a+b)^2 = a^2 + 2ab + b^2$

4. $(-a+b)^2 = [-(a-b)]^2 = (a-b)^2 = a^2 - 2ab + b^2$

Combining 1 and 3, we have :

5. $(\pm a \pm b)^2 = a^2 + 2ab + b^2$, note that the signs of a and b are **the same** and the 2ab term is positive.

Combining 2 and 4, we have :

6. $(\pm a \mp b)^2 = a^2 - 2ab + b^2$, note that the signs of a and b are **opposite**. and the 2ab term is negative.

Also, the sign of a^2 and b^2 must always be **positive** in 4 or 5.

Examples: Pay attention to the signs of the expanded form.

1. $\left(-\sqrt{x} + 2\sqrt{y}\right)^2 = \left(\sqrt{x}\right)^2 - 2\left(\sqrt{x}\right)\left(2\sqrt{y}\right) + \left(2\sqrt{y}\right)^2 = x - 4\sqrt{xy} + 4y = \underline{\underline{x + 4y - 4\sqrt{xy}}}$

2. $-\frac{1}{2t}\left(-\frac{2t}{3} - \frac{3}{2t}\right)^2 = -\frac{1}{2t}\left[\left(\frac{2t}{3}\right)^2 + 2\left(\frac{2t}{3}\right)\left(\frac{3}{2t}\right) + \left(\frac{3}{2t}\right)^2\right] = -\frac{1}{2t}\left[\frac{4t^2}{9} + 2 + \frac{9}{4t^2}\right]$

$= \underline{\underline{-\frac{2t}{9} - \frac{1}{t} - \frac{9}{8t^3}}}$

3. $\left(\frac{1}{a} + \frac{2}{b}\right)^2 + \left(\frac{1}{a} - \frac{2}{b}\right)^2 + \left(-\frac{2}{a} - \frac{1}{b}\right)^2 + \left(-\frac{2}{a} + \frac{1}{b}\right)^2$

$= \left[\frac{1}{a^2} + \frac{4}{ab} + \frac{4}{b^2}\right] + \left[\frac{1}{a^2} - \frac{4}{ab} + \frac{4}{b^2}\right] + \left[\frac{4}{a^2} + \frac{4}{ab} + \frac{1}{b^2}\right] + \left[\frac{4}{a^2} - \frac{4}{ab} + \frac{1}{b^2}\right]$

$= \underline{\underline{\frac{10}{a^2} + \frac{10}{b^2}}}$

Binomial with higher power

7. $(a+b)^3 = (a+b)^2(a+b) = (a^2+2ab+b^2)(a+b) = a^3+3a^2b+3ab^2+b^3$

8. $(a-b)^3 = (a-b)^2(a-b) = (a^2-2ab+b^2)(a-b) = a^3-3a^2b+3ab^2-b^3$

9. $(-a-b)^3 = -(a+b)^3 = -a^3-3a^2b-3ab^2-b^3$

10. $(-a+b)^3 = -(a-b)^3 = -a^3+3a^2b-3ab^2+b^3$

11. If n is **even**, we have : $(-a-b)^n = (a+b)^n$

and $(-a+b)^n = (a-b)^n$

12. If n is **odd**, we have : $(-a-b)^n = -(a+b)^n$

and $(-a+b)^n = -(a-b)^n$